Problemset 2

Exercise 1

(1) Define the characteristic function F_n which takes a whole number as argument and delivers 1 als value iff the whole number is also a natural number.

Answer $F_n :=$ $f : \mathbb{Z} \to \{0, 1\}$
For all $x \in \mathbb{Z}, f(x) = 1$ iff $x \in \mathbb{N}$

(2) What is the set characterized by $F_{\leq 4}$?
$F_{\leq 4} :=$ $f : \mathbb{N} \to \{0, 1\}$
For all $x \in \mathbb{N}, f(x) = 1$ iff $x \leq 4$

Answer $char_{F_{\leq 4}} = \{1, 2, 3, 4\}$

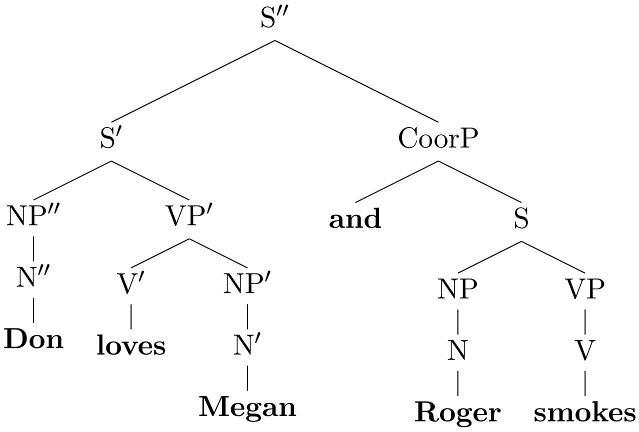
Exercise 2

Assume that (3) has the structure in (4). In contrast to the first problem set all phrase structure trees are now binary branching. (i) Define a semantic value for every new lexical entry. (ii) The lexical entry for **and** from the first problem set cannot be used anymore because of the binary branching. Define a new one. (iii) Modify, if necessary, the inventory of denotation types. (iv) The old rule for structures with **and** cannot be used anymore either because of binary branching. Formulate a new one so that CoorP can be interpreted. Note that semantic composition is based on function application. (v) Calculate with the help of the results from (i)-(iv) the truth-conditions of (4) step by step.

Hint: Binary branching makes it necessary that **and** now denote a function-valued function.

(3) **Don loves Megan and Roger smokes.**

(4)



Answer

(i) Lexical entries

(5) a. $[\![\textbf{Don}]\!] =$ Don
b. $[\![\textbf{Roger}]\!] =$ Roger

c. $[\![\textbf{Megan}]\!]$ = Megan

d. $[\![\textbf{loves}]\!] =$ $f : D \to \{g : g$ is a function from D to $\{0,1\}\}$
For all $x, y \in D, f(x)(y) = 1$ iff y loves x

e. $[\![\textbf{smokes}]\!] =$ $f : D \to \{0,1\}$
For all $x \in D, f(x) = 1$ iff x smokes

f. $[\![\textbf{and}]\!] =$ $f : D_t \to \{g : g$ is a function from D_t to $D_t\}$
for all $x, y \in D_t, f(x)(y) = 1$ iff $y = x = 1$

(ii) Denotation types

(6) a. Elements of D_e
b. Elements of D_t
c. Elements of $D_{\langle e,t\rangle}$
d. Elements of $D_{\langle e,\langle e,t\rangle\rangle}$
e. Elemente of $D_{\langle t,t\rangle}$
f. Elemente of $D_{\langle t,\langle t,t\rangle\rangle}$

(iii) Semantic rules

S1 If α has the form $[_{\text{S}}\ \beta\ \gamma]$, then $[\![\alpha]\!] = [\![\gamma]\!]([\![\beta]\!])$.

S2 If α has the form $[_{\text{NP}}\ \beta]$, then $[\![\alpha]\!] = [\![\beta]\!]$.

S3 If α has the form $[_{\text{VP}}\ \beta]$, then $[\![\alpha]\!] = [\![\beta]\!]$.

S4 If α has the form $[_{\text{N}}\ \beta]$, then $[\![\alpha]\!] = [\![\beta]\!]$.

S5 If α has the form $[_{\text{V}}\ \beta]$, then $[\![\alpha]\!] = [\![\beta]\!]$.

S7 If α has the form $[_{\text{VP}}\ \beta\ \gamma]$, then $[\![\alpha]\!] = [\![\beta]\!]([\![\gamma]\!])$.

S8 If α is of the form $[_{\text{CoorP}}\ \beta\ \gamma]$, then $[\![\alpha]\!] = [\![\beta]\!]([\![\gamma]\!])$.

(iv) Wahrheitsbedingungen

$$
\begin{aligned}
[\![\textbf{S}'']\!] &= [\![\textbf{CoorP}]\!]([\![\textbf{S}']\!]) && \text{(S1)}\\
&= [\![\textbf{and}]\!]([\![\textbf{S}]\!])([\![\textbf{S}']\!]) && \text{(S8)}\\
&= [\![\textbf{and}]\!]([\![\textbf{VP}]\!]([\![\textbf{NP}]\!]))([\![\textbf{VP}']\!]([\![\textbf{NP}'']\!])) && (2 \times \text{S1})\\
&= [\![\textbf{and}]\!]([\![\textbf{VP}]\!]([\![\textbf{NP}]\!]))([\![\textbf{V}']\!]([\![\textbf{NP}']\!])([\![\textbf{NP}'']\!])) && \text{(S7)}\\
&= [\![\textbf{and}]\!]([\![\textbf{VP}]\!]([\![\textbf{N}]\!]))([\![\textbf{V}']\!]([\![\textbf{N}']\!])([\![\textbf{N}'']\!])) && (3 \times \text{S2})\\
&= [\![\textbf{and}]\!]([\![\textbf{VP}]\!]([\![\textbf{Roger}]\!]))([\![\textbf{V}']\!]([\![\textbf{Megan}]\!])([\![\textbf{Don}]\!])) && (3 \times \text{S4})
\end{aligned}
$$

$$= [\![\textbf{and}]\!]([\![\textbf{V}]\!]([\![\textbf{Roger}]\!]))([\![\textbf{V}']\!]([\![\textbf{Megan}]\!])([\![\textbf{Don}]\!])) \quad \text{(S3)}$$

$$= [\![\textbf{and}]\!]([\![\textbf{smokes}]\!]([\![\textbf{Roger}]\!]))([\![\textbf{loves}]\!]([\![\textbf{Megan}]\!])([\![\textbf{Don}]\!])) \quad (2 \times \text{S5})$$

$$= \left[\begin{array}{l} f : D_t \rightarrow \{g : g \text{ is a function from } D_t \text{ to } D_t\} \\ \text{for all } x, y \in D_t, f(x)(y) = 1 \text{ iff } y = x = 1 \end{array}\right]$$
$$([\![\textbf{smokes}]\!]([\![\textbf{Roger}]\!]))([\![\textbf{loves}]\!]([\![\textbf{Megan}]\!])([\![\textbf{Don}]\!])) \quad \text{(lexicon)}$$

Fact 1: $\left[\begin{array}{l} f : D_t \rightarrow \{g : g \text{ is a function from } D_t \text{ to } D_t\} \\ \text{for all } x, y \in D_t, f(x)(y) = 1 \text{ iff } y = x = 1 \end{array}\right]$
$([\![\textbf{smokes}]\!]([\![\textbf{Roger}]\!]))([\![\textbf{loves}]\!]([\![\textbf{Megan}]\!])([\![\textbf{Don}]\!])) = 1$ iff
$[\![\textbf{loves}]\!]([\![\textbf{Megan}]\!])([\![\textbf{Don}]\!]) = [\![\textbf{smokes}]\!]([\![\textbf{Roger}]\!]) = 1$

$$[\![\textbf{S}'']\!] = 1 \text{ iff } [\![\textbf{loves}]\!]([\![\textbf{Megan}]\!])([\![\textbf{Don}]\!]) = [\![\textbf{smokes}]\!]([\![\textbf{Roger}]\!]) = 1 \quad \text{(fact 1)}$$

$$= 1 \text{ iff } [\![\textbf{loves}]\!](\text{Megan})(\text{Don}) = [\![\textbf{smokes}]\!](\text{Roger}) = 1 \quad (3 \times \text{lexicon})$$

$$= 1 \text{ iff } \left[\begin{array}{l} f : D \rightarrow \{g : g \text{ is a function from } D \text{ to } \{0,1\}\} \\ \text{For all } x, y \in D, f(x)(y) = 1 \text{ iff } y \text{ loves } x \end{array}\right](\text{Megan})(\text{Don}) =$$
$$\left[\begin{array}{l} f : D \rightarrow \{0,1\} \\ \text{For all } x \in D, f(x) = 1 \text{ iff } x \text{ smokes} \end{array}\right](\text{Roger}) = 1$$
$$(2 \times \text{lexicon})$$

Fact 2: $\left[\begin{array}{l} f : D \rightarrow \{0,1\} \\ \text{For all } x \in D, f(x) = 1 \text{ iff } x \text{ smokes} \end{array}\right](\text{Roger}) = 1$ iff Roger smokes

Fact 3: $\left[\begin{array}{l} f : D \rightarrow \{g : g \text{ is a function from } D \text{ to } \{0,1\}\} \\ \text{For all } x, y \in D, f(x)(y) = 1 \text{ iff } y \text{ loves } x \end{array}\right](\text{Megan})(\text{Don}) = 1$ iff Don loves Megan

$$[\![\textbf{S}'']\!] = 1 \text{ iff Don loves Megan and Roger smokes} \quad \text{(facts 2,3)}$$