Problemset 1 'Introduction to semantics'

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(1) Which of the following is true, which false (and under which conditions)?

a. $\{a, b, c\} = \{c, b, a\}$
b. $\{a, b\} \in \{a, b, c, d\}$
c. $\{a, b, c\} = \{a, b, c, d\}$
d. $\{x : x \in \{y : y \neq 4\}\} = \{y : y \in \{x : x \neq 4\}\}$
e. $\{x : \{y : y \text{ loves } x\} = \{\text{Maria}\}\} = \{x : \{y : x \text{ loves } y\} = \{\text{Maria}\}\}$
f. $\{x : x \in A\} \subseteq A$
g. $\{x : x \text{ loves } b\} = \{x : x \text{ loves } a\}$

(2) Define ...

a. as A the set whose elements are all the numbers between 2 and 10 and nothing else.
b. as B the set whose elements are all the students who do not read anything and nothing else.
c. as C the set whose elements are all the students who are not loved by anyone and nothing else.

(3) Define the function F_I which takes a French president as argument and maps the president to his year of birth.

(4) Define domain and range of the following functions:

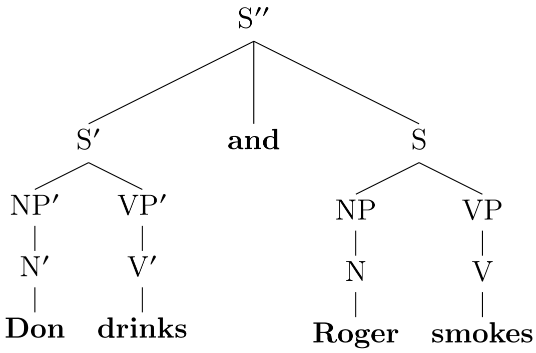
a. $F_1 := \{\langle x, y\rangle : x \text{ is a car and } y \text{ is } x\text{'s manufacturer}\}$
b. $F_2 := \{\langle x, y\rangle : x \text{ is an object and } y \text{ is } x\text{'s hair color}\}$

(5) Assume the structure in (b) for the sentence in (a). (i) Define an extension/denotation for every new lexical expression. (ii) Add if necessary new denotation types to the inventory of possible denotation types. (iii) Formulate for those non-terminal nodes for which the present system cannot assign a denotation yet, a suitable semantic interpretation rule. These rules should be based on function application. (iv) Calculate with the help of your results from (i)-(iii) the truth-condtiions of (b) step by step.

Hint: the principle of compositionality requires that **and** denote a binary function.

(a) **Don drinks and Roger smokes.**

(b)



Solutions

(1) a. $\{a, b, c\} = \{c, b, a\}$ **true**

b. $\{a, b\} \in \{a, b, c, d\}$ **false**

c. $\{a, b, c\} = \{a, b, c, d\}$ **false**

d. $\{x : x \in \{y : y \neq 4\}\} = \{y : y \in \{x : x \neq 4\}\}$ **true**

e. $\{x : \{y : y \text{ loves } x\} = \{\text{Maria}\}\} = \{x : \{y : x \text{ loves } y\} = \{\text{Maria}\}\}$ **true, if the set of those only loved by by Mary is equivalent to the set of those only loving Mary**

f. $\{x : x \in A\} \subseteq A$ **true**

g. $\{x : x \text{ loves } b\} = \{x : x \text{ loves } a\}$ **true, iff** $a = b$

(2) a. **Depends a bit on whether 'between' is interpreted inclusively or exclusively:**
$A := \{3, 4, 5, 6, 7, 8, 9\}$
$A := \{x : x \geq 2$ **and** $x \leq 10\}$

b. $B := \{x : x$ **is a student and** x **does not read anything**$\}$
$B := \{x : x$ **is a student and** $\{y : x$ **reads** $y\} = \varnothing\}$

c. $C := \{x : x$ **is a student and no one loves** $x\}$
$C := \{x : x$ **is a student and** $\{y : y$ **loves** $x\} = \varnothing\}$

(3) a. $F_I := \{\langle x, y\rangle : x \text{ is a French president and } y \text{ is } x\text{'s year of birth}\}$

b. Let F_I be that function f such that
$f : \{x : x \text{ is a French president}\} \rightarrow \mathbb{N}$, and for every $x \in \{x : x \text{ is a French president}\}$, $f(x) =$ the year in which x was born.

c. $F_I :=$ $f : \{x : x \text{ is a French president}\} \rightarrow \mathbb{N}$
For every $x \in \{x : x \text{ is a French president}\}$, $f(x) =$ the year x was born.

(4) a. domain of $F_1 = \{x : x \text{ is a car}\}$

b. range of $F_1 = \{x : x \text{ manufactures cars}\}$

c. domain of $F_2 = \{x : x \text{ is a mammal}\}$

d. range of $F_2 = \{x : x \text{ is a color that some mammal's hair has}\}$

Solution to (5)

(i) Lexical entries

(6)
a. $[\![\text{Don}]\!]^s = \text{Don}$
b. $[\![\text{Roger}]\!]^s = \text{Roger}$
c. $[\![\text{drinks}]\!]^s = \begin{array}{l} f : D \to \{0,1\} \\ \text{For all } x \in D, f(x) = 1 \text{ iff } x \text{ drinks in } s \end{array}$
d. $[\![\text{smokes}]\!]^s = \begin{array}{l} f : D \to \{0,1\} \\ \text{For all } x \in D, f(x) = 1 \text{ iff } x \text{ smokes in } s \end{array}$
e. $[\![\text{and}]\!]^s = \begin{array}{l} f : \{0,1\} \times \{0,1\} \to \{0,1\} \\ \text{for all } x, y \in \{0,1\}, f(x,y) = 1 \text{ iff } x = y = 1 \end{array}$

(ii) Denotation types

(7)
a. Functions from $\{0,1\}$ to $\{0,1\}$
b. Functions from $\{0,1\} \times \{0,1\}$ to $\{0,1\}$ (functions from pairs of truth-values to truth-values))

(iii) Semantic rules

S6 If α has the form [S **not** β], then $[\![\alpha]\!]^s = [\![\textbf{not}]\!]^s([\![\beta]\!]^s)$.

S7 If α is of the form [S β **and** γ], then $[\![\alpha]\!]^s = [\![\textbf{and}]\!]^s([\![\beta]\!]^s, [\![\gamma]\!]^s)$.

(iv) Wahrheitsbedingungen

$$
\begin{array}{lll}
[\![\text{S}'']\!]^s & = [\![\text{and}]\!]^s([\![\text{S}']\!]^s, [\![\text{S}]\!]^s) & \text{(S7)} \\
& = [\![\text{and}]\!]^s([\![\text{VP}']\!]^s([\![\text{NP}']\!]^s), [\![\text{VP}]\!]^s([\![\text{NP}]\!]^s)) & 2\times\text{(S1)} \\
& = [\![\text{and}]\!]^s([\![\text{VP}']\!]^s([\![\text{N}']\!]^s), [\![\text{VP}]\!]^s([\![\text{N}]\!]^s)) & 2\times\text{(S2)} \\
& = [\![\text{and}]\!]^s([\![\text{VP}']\!]^s([\![\text{Don}]\!]^s), [\![\text{VP}]\!]^s([\![\text{Roger}]\!]^s)) & 2\times\text{(S4)} \\
& = [\![\text{and}]\!]^s([\![\text{V}']\!]^s([\![\text{Don}]\!]^s), [\![\text{V}]\!]^s([\![\text{Roger}]\!]^s)) & 2\times\text{(S3)} \\
& = [\![\text{and}]\!]^s([\![\text{drinks}]\!]^s([\![\text{Don}]\!]^s), [\![\text{smokes}]\!]^s([\![\text{Roger}]\!]^s)) & 2\times\text{(S5)} \\
& = \left[\begin{array}{l} f : \{0,1\} \times \{0,1\} \to \{0,1\} \\ \text{for all } x, y \in \{0,1\}, f(x,y) = 1 \text{ iff } x = y = 1 \end{array} \right] & \\
& \quad ([\![\text{drinks}]\!]^s([\![\text{Don}]\!]^s), [\![\text{smokes}]\!]^s([\![\text{Roger}]\!]^s)) & \text{(lexicon)}
\end{array}
$$

Fact 1 $\left[\begin{array}{l} f : \{0,1\} \times \{0,1\} \to \{0,1\} \\ \text{for all } x, y \in \{0,1\}, f(x,y) = 1 \text{ iff } x = y = 1 \end{array} \right]$
$([\![\text{drinks}]\!]^s([\![\text{Don}]\!]^s), [\![\text{smokes}]\!]^s([\![\text{Roger}]\!]^s)) = 1$ iff
$[\![\text{drinks}]\!]^s([\![\text{Don}]\!]^s) = [\![\text{smokes}]\!]^s([\![\text{Roger}]\!]^s) = 1$

$$[\![\text{S}'']\!]^s = 1 \text{ iff } [\![\text{drinks}]\!]^s([\![\text{Don}]\!]^s) = [\![\text{smokes}]\!]^s([\![\text{Roger}]\!]^s) = 1 \qquad \text{(fact 1)}$$

$$= 1 \text{ iff } [\![\text{drinks}]\!]^s(\text{Don}) = [\![\text{smokes}]\!]^s(\text{Roger}) = 1 \qquad 2\times(\text{lexicon})$$

$$= 1 \text{ iff } \begin{bmatrix} f : D \to \{0,1\} \\ \text{For all } x \in D, f(x) = 1 \text{ iff } x \text{ drinks in } s \end{bmatrix}(\text{Don}) = \begin{bmatrix} f : D \to \{0,1\} \\ \text{For all } x \in D, f(x) = 1 \text{ iff } x \text{ smokes in } s \end{bmatrix}(\text{Roger}) = 1 \qquad 2\times(\text{lexicon})$$

Fact 2: $\begin{bmatrix} f : D \to \{0,1\} \\ \text{For all } x \in D, f(x) = 1 \text{ iff } x \text{ drinks in } s \end{bmatrix}(\text{Don}) = 1 \text{ iff Don drinks in } s$

Fact 3: $\begin{bmatrix} f : D \to \{0,1\} \\ \text{For all } x \in D, f(x) = 1 \text{ iff } x \text{ smokes in } s \end{bmatrix}(\text{Roger}) = 1 \text{ iff Roger smokes in } s$

$$[\![\text{S}'']\!]^s = 1 \text{ iff Don drinks in } s \text{ and Roger smokes in } s \qquad (\text{facts 2, 3})$$