Introduction to semantics
Heim & Kratzer 1998, chapter 3

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July 26, 2016

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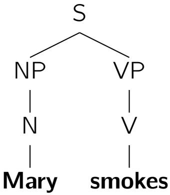
Overview

1. What we did yesterday
2. How to expand the system
3. Sets and their characteristic functions
4. Transitive verbs
5. Semantic types

Proof of truth-conditions I



Assume $[\![[_S \ldots]]\!] = 1$ iff Mary smokes

S1 If α has the form

, then $[\![\alpha]\!] = [\![\gamma]\!]([\![\beta]\!])$.

$[\![[_S \ldots]]\!] = [\![[_{VP} \ldots]]\!]([\![[_{NP} \ldots]]\!])$

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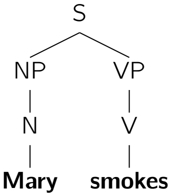

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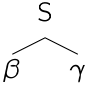

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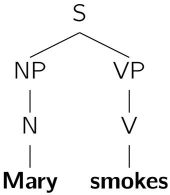

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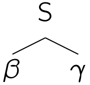

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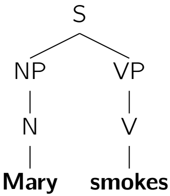

S2 If α has the form NP, then $[\![\alpha]\!] = [\![\beta]\!]$.
$\qquad\qquad\qquad\qquad\qquad\quad|$
$\qquad\qquad\qquad\qquad\qquad\quad\beta$

$[\![[_S \ldots]]\!] = [\![[_{VP} \ldots]]\!]([\![[_N \ldots]]\!])$

S4 If α has the form N, then $[\![\alpha]\!] = [\![\beta]\!]$.
$\qquad\qquad\qquad\qquad\qquad\quad|$
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$[\![[_S \ldots]]\!] = [\![[_{VP} \ldots]]\!]([\![\textbf{Mary}]\!])$

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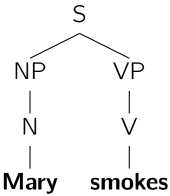

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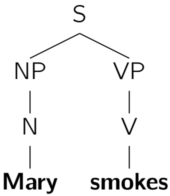

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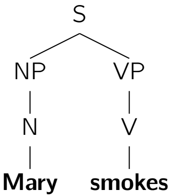

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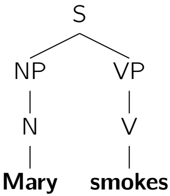

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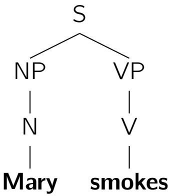
Proof of truth-conditions III



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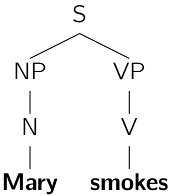

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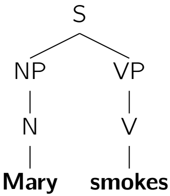

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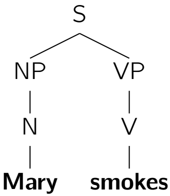

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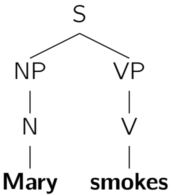

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$[\![\textbf{Mary}]\!] = \text{Mary}$ lexicon

$[\![\textbf{smokes}]\!] = f : D \rightarrow \{0, 1\}$
For all $x \in D$, $f(x) = 1$ iff x smokes lexicon

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End of proof

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End of proof

Truth-conditions of a sentence

Lexicon defines extension of an intransitive verb as a condition

$[\![\textbf{smokes}]\!] = f : D \rightarrow \{0, 1\}$
For all $x \in D$, $f(x) = 1$ iff x smokes

Together with the rules we get truth-conditions as meaning of S .

This seems adequate regarding the intuitions of speakers with respect to the meaning of $[\![\textbf{smokes}]\!]$.

Normally we do not have full information about who smokes.

Truth-value of a sentence

Assume $D = \{\text{John, Mary, Sue}\}$ in Situation s

Extension of **smokes** represented by table

$$[\![\textbf{smokes}]\!] = \begin{bmatrix} \text{John} & \rightarrow & 0 \\ \text{Mary} & \rightarrow & 1 \\ \text{Sue} & \rightarrow & 0 \end{bmatrix}$$

Delivers as denotation of S a truth-value

$$[\![[_S \ldots]]\!] = \begin{bmatrix} \text{John} & \rightarrow & 0 \\ \text{Mary} & \rightarrow & 1 \\ \text{Sue} & \rightarrow & 0 \end{bmatrix}(\text{Mary}) = 1$$

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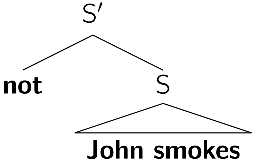
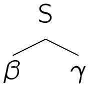
Negation

We cannot assign a denotation to (1)

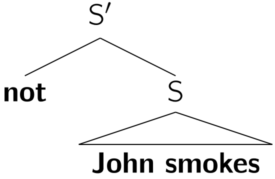
(1) **It is not the case that John smokes.**



S1 is not applicable: $[\![\gamma]\!] = [\![[_S$ **John smokes** $]]\!]$ is not a function. It cannot be applied to $[\![$**not**$]\!]$

If α has the form  , then $[\![\alpha]\!] = [\![\gamma]\!]([\![\beta]\!])$. (S1)

Modifications of the interpretative system



According to Frege's conjecture function application combines $[\![\textbf{not}]\!]$ with $[\![\textbf{S}]\!]$.

Since, $[\![\textbf{S}]\!]$ is not a function, $[\![\textbf{not}]\!]$ applied as function to $[\![\textbf{S}]\!]$.

This necessitates a new denotation type, a new lexical entry, a new semantic rule.

Inventory of denotation types

- Elements of D: the set of actual individuals
- Elements of $\{0, 1\}$: the set of truth-values
- Functions from D to $\{0, 1\}$
- Functions from $\{0, 1\}$ to $\{0, 1\}$

Lexical entries

- Proper names

  $[\![\textbf{Mary}]\!]$ = Mary
  $[\![\textbf{Bill}]\!]$ = Bill
  ...

- Intransitive verbs

  $[\![\textbf{dances}]\!] = f : D \to \{0,1\}$
  For all $x \in D$, $f(x) = 1$ iff x dances
  $[\![\textbf{smokes}]\!] = f : D \to \{0,1\}$
  For all $x \in D$, $f(x) = 1$ iff x smokes

- Operators, connectives

  $[\![\textbf{not}]\!] = f : \{0,1\} \to \{0,1\}$
  For all $x \in \{0,1\}$, $f(x) = 1$ iff $x = 0$

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Semantic rules

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S2 If α has the form $\begin{array}{c} \text{NP} \\ | \\ \beta \end{array}$, then $[\![\alpha]\!] = [\![\beta]\!]$.

S3 If α has the form $\begin{array}{c} \text{VP} \\ | \\ \beta \end{array}$, then $[\![\alpha]\!] = [\![\beta]\!]$.

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S5 If α has the form $\begin{array}{c} \text{V} \\ | \\ \beta \end{array}$, then $[\![\alpha]\!] = [\![\beta]\!]$.

S6 If α has the form $\begin{array}{c} \text{S} \\ \textbf{not} \quad \beta \end{array}$, then $[\![\alpha]\!] = [\![\textbf{not}]\!]([\![\beta]\!])$.

Semantic rules

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Truth-conditions I

Assumption: $[\![[_{S'} \ldots]]\!] = 1$ iff John does not smoke

$$
\begin{aligned}
[\![[_{S'} \ldots]]\!] &= [\![\textbf{not}]\!]([\![\textbf{S}]\!]) && \text{(S6)}\\
&= [\![\textbf{not}]\!]([\![[_{VP} \ldots]]\!]([\![[_{NP} \ldots]]\!])) && \text{(S1)}\\
&= [\![\textbf{not}]\!]([\![[_{VP} \ldots]]\!]([\![[_{N} \ldots]]\!])) && \text{(S2)}\\
&= [\![\textbf{not}]\!]([\![[_{VP} \ldots]]\!]([\![\textbf{John}]\!])) && \text{(S4)}\\
&= [\![\textbf{not}]\!]([\![[_{VP} \ldots]]\!](\text{John})) && \text{(lexicon)}\\
&= [\![\textbf{not}]\!]([\![[_{V} \ldots]]\!](\text{John})) && \text{(S3)}\\
&= [\![\textbf{not}]\!]([\![\textbf{smokes}]\!](\text{John})) && \text{(S5)}\\
&= \\
&[\![\text{not}]\!]\left(\begin{bmatrix} f : D \to \{0,1\} \\ \text{For all } x \in D, f(x) = 1 \text{ iff } x \text{ smokes in } s \end{bmatrix}(\text{John})\right) \\
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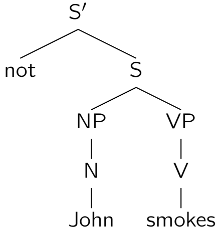

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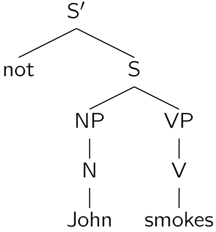

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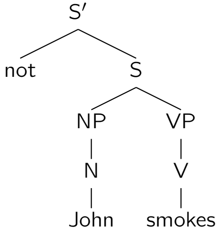
[\![[_{S'} \ldots]]\!] &= [\![\textbf{not}]\!]([\![\textbf{S}]\!]) && \text{(S6)}\\
&= [\![\textbf{not}]\!]([\![[_{VP} \ldots]]\!]([\![[_{NP} \ldots]]\!])) && \text{(S1)}\\
&= [\![\textbf{not}]\!]([\![[_{VP} \ldots]]\!]([\![[_{N} \ldots]]\!])) && \text{(S2)}\\
&= [\![\textbf{not}]\!]([\![[_{VP} \ldots]]\!]([\![\textbf{John}]\!])) && \text{(S4)}\\
&= [\![\textbf{not}]\!]([\![[_{VP} \ldots]]\!](\text{John})) && \text{(lexicon)}\\
&= [\![\textbf{not}]\!]([\![[_{V} \ldots]]\!](\text{John})) && \text{(S3)}\\
&= [\![\textbf{not}]\!]([\![\textbf{smokes}]\!](\text{John})) && \text{(S5)}\\
&= \\
&[\![\text{not}]\!]\left(\left[\begin{array}{l} f : D \to \{0,1\} \\ \text{For all } x \in D, f(x) = 1 \text{ iff } x \text{ smokes in } s \end{array}\right](\text{John})\right) \\
& && \text{(lexicon)}
\end{aligned}
$$



Truth-conditions I

Assumption: $[\![[_{S'} \ldots]]\!] = 1$ iff John does not smoke

$$
\begin{aligned}
[\![[_{S'} \ldots]]\!] &= [\![\textbf{not}]\!]([\![\textbf{S}]\!]) && \text{(S6)} \\
&= [\![\textbf{not}]\!]([\![[_{VP} \ldots]]\!]([\![[_{NP} \ldots]]\!])) && \text{(S1)} \\
&= [\![\textbf{not}]\!]([\![[_{VP} \ldots]]\!]([\![[_{N} \ldots]]\!])) && \text{(S2)} \\
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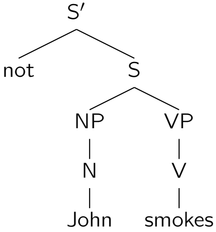
\end{aligned}
$$



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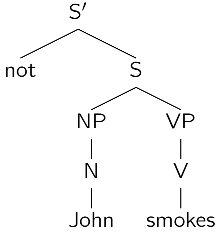

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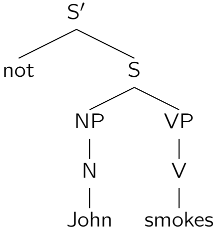

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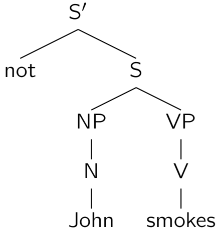

Truth-conditions II

$$[\![[_{S'} \ldots]]\!] = [\![\textbf{not}]\!] \left(\left[\begin{array}{l} f : D \to \{0,1\} \\ \text{For all } x \in D, f(x) = 1 \text{ iff } x \text{ smokes in } s \end{array} \right] (\text{John}) \right)$$

$$= \left[\begin{array}{l} f : \{0,1\} \to \{0,1\} \\ \text{for all } x \in \{0,1\}, f(x) = 1 \text{ iff } x = 0 \end{array} \right]$$

$$\left(\left[\begin{array}{l} f : D \to \{0,1\} \\ \text{For all } x \in D, f(x) = 1 \text{ iff } x \text{ smokes in } s \end{array} \right] (\text{John}) \right) \qquad \text{(lexicon)}$$

$$= 1 \text{ iff } \left[\begin{array}{l} f : D \to \{0,1\} \\ \text{For all } x \in D, f(x) = 1 \text{ iff } x \text{ smokes in } s \end{array} \right] (\text{John}) = 0$$

$= 1$ iff John does not smoke

Overview

1. What we did yesterday
2. How to expand the system
3. Sets and their characteristic functions
4. Transitive verbs
5. Semantic types

Intransitive verbs as sets

Intransitive verbs as functions from individuals to truth-values

$$[\![\textbf{smokes}]\!] = f : D \rightarrow \{0, 1\}$$
$$\text{For all } x \in D, f(x) = 1 \text{ iff } x \text{ smokes}$$

Intransitive verbs can also be seen as sets of individuals

$$[\![\textbf{smokes}]\!] = \{x : x \in D \text{ and } x \text{ smokes } \}$$
abbreviated as: $[\![\textbf{smokes}]\!] = \{x \in D : x \text{ smokes } \}$

This would require an alternative to S1:

S1' If α has the form $[_S\ \beta\ \gamma]$, then $[\![\alpha]\!] = 1$ iff $[\![\beta]\!] \in [\![\gamma]\!]$.

Intransitive verbs as sets

Intransitive verbs as functions from individuals to truth-values

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Intransitive verbs as sets

Intransitive verbs as functions from individuals to truth-values

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This would require an alternative to S1:

S1' If α has the form $[_S\ \beta\ \gamma]$, then $[\![\alpha]\!] = 1$ iff $[\![\beta]\!] \in [\![\gamma]\!]$.

Relation between sets and their characteristic functions

- Let A be a set. Then cha_A is that function f such that for every $x \in A$, $f(x) = 1$ and for every $x \notin A$, $f(x) = 0$. $char_A$ is the characteristic function of A.

- Let f be a function whose range is $\{0, 1\}$. Then $char_f$ is the set $\{x \in D : f(x) = 1\}$. $char_f$ is the set characterized by f.

Sets and their characteristic functions

Assume $D = \{\text{John, Mary, Ann}\}$

Sets

- $[\![\textbf{smoke}]\!] = \{\text{Mary, Ann}\}$
- $[\![\textbf{dance}]\!] = \{\text{Mary}\}$

Characteristic functions

- $[\![\textbf{smoke}]\!] = \begin{bmatrix} \text{John} & \to & 0 \\ \text{Mary} & \to & 1 \\ \text{Ann} & \to & 1 \end{bmatrix}$
- $[\![\textbf{dance}]\!] = \begin{bmatrix} \text{John} & \to & 0 \\ \text{Mary} & \to & 1 \\ \text{Ann} & \to & 0 \end{bmatrix}$

Questions about sets and characteristic functions

Q1 $[\![\textbf{sleep}]\!] = \begin{bmatrix} \text{Maria} & \rightarrow & 0 \\ \text{Anton} & \rightarrow & 1 \\ \text{Susi} & \rightarrow & 1 \end{bmatrix}$. What is $char_{[\![\text{sleep}]\!]}$? Why?

A1 $char_{[\![\text{sleep}]\!]} = \{\text{Anton, Susi}\}$, because $\{x : [\![\textbf{sleep}]\!](x) = 1\} = \{\text{Anton, Susi}\}$

Q2 $D = \{\text{Peter, Hans, Klara}\}$ and $[\![\textbf{drive}]\!] = \{\text{Peter}\}$. What is $char_{[\![\text{drive}]\!]}$?

A2 $char_{[\![\text{drive}]\!]} = \begin{bmatrix} \text{Peter} & \rightarrow & 1 \\ \text{Hans} & \rightarrow & 0 \\ \text{Klara} & \rightarrow & 0 \end{bmatrix}$

Questions about sets and characteristic functions

Q1 $[\![\textbf{sleep}]\!] = \begin{bmatrix} \text{Maria} & \rightarrow & 0 \\ \text{Anton} & \rightarrow & 1 \\ \text{Susi} & \rightarrow & 1 \end{bmatrix}$. What is $char_{[\![\text{sleep}]\!]}$? Why?

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Overview

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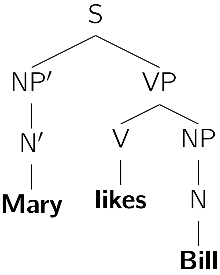
Missing rule and denotation



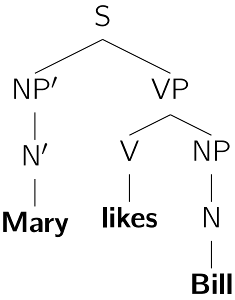
We need a new rule for function application of ⟦**V**⟧ to ⟦**NP**⟧.

S5 tells us **V** inherits denotation of **likes**.

likes denotes not a function from individuals to truth-values.

Otherwise the new rule would give a truth-value for ⟦**VP**⟧, which cannot be applied to ⟦**NP′**⟧.

Branching VP-nodes

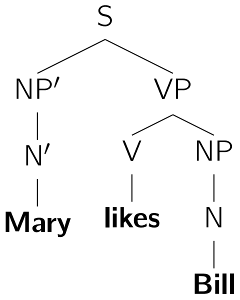


S denotes a truth-value, NP′ and NP denote individuals.

VP denotes function from individuals to truth-values.

likes and **V** denote a function from individuals to functions from individuals to truth-values. They denote function-valued functions.

Branching VP-nodes



S denotes a truth-value, NP′ and NP denote individuals.

VP denotes function from individuals to truth-values.

likes and **V** denote a function from individuals to functions from individuals to truth-values. They denote function-valued functions.

Function-valued functions I

$D = \{\text{Anna, Berta, Clara}\}$

(2) maps every $x \in D$ to a function.

Each of these functions maps every $x \in D$ to a truth-value.

$$[\![\textbf{likes}]\!] = \begin{bmatrix} \text{Anna} & \rightarrow & \begin{bmatrix} \text{Anna} & \rightarrow & 1 \\ \text{Berta} & \rightarrow & 1 \\ \text{Clara} & \rightarrow & 1 \end{bmatrix} \\ \text{Berta} & \rightarrow & \begin{bmatrix} \text{Anna} & \rightarrow & 0 \\ \text{Berta} & \rightarrow & 1 \\ \text{Clara} & \rightarrow & 0 \end{bmatrix} \\ \text{Clara} & \rightarrow & \begin{bmatrix} \text{Anna} & \rightarrow & 1 \\ \text{Berta} & \rightarrow & 1 \\ \text{Clara} & \rightarrow & 0 \end{bmatrix} \end{bmatrix}$$

Function-valued functions II

(2) maps every $x \in D$ to the (characteristic) function of the set of individuals in D who like x.

(2) maps Anna to a function f which maps Berta to 1.

This means for us that Berta likes Anna.

$$[\![\textbf{likes}]\!] = \begin{bmatrix} \text{Anna} & \rightarrow & \begin{bmatrix} \text{Anna} & \rightarrow & 1 \\ \text{Berta} & \rightarrow & 1 \\ \text{Clara} & \rightarrow & 1 \end{bmatrix} \\ \text{Berta} & \rightarrow & \begin{bmatrix} \text{Anna} & \rightarrow & 0 \\ \text{Berta} & \rightarrow & 1 \\ \text{Clara} & \rightarrow & 0 \end{bmatrix} \\ \text{Clara} & \rightarrow & \begin{bmatrix} \text{Anna} & \rightarrow & 1 \\ \text{Berta} & \rightarrow & 1 \\ \text{Clara} & \rightarrow & 0 \end{bmatrix} \end{bmatrix}$$

Questions about function-valued functions

$$[\![\textbf{likes}]\!] = \begin{bmatrix} \text{Anna} & \to & \begin{bmatrix} \text{Anna} & \to & 1 \\ \text{Berta} & \to & 1 \\ \text{Clara} & \to & 1 \end{bmatrix} \\ \text{Berta} & \to & \begin{bmatrix} \text{Anna} & \to & 0 \\ \text{Berta} & \to & 1 \\ \text{Clara} & \to & 0 \end{bmatrix} \\ \text{Clara} & \to & \begin{bmatrix} \text{Anna} & \to & 1 \\ \text{Berta} & \to & 1 \\ \text{Clara} & \to & 0 \end{bmatrix} \end{bmatrix}$$

Q1 Who does Anna like, who does she not like?

A1 Anna likes Anna and Clara, but not Berta.

Q2 Who likes everyone?

A2 Berta

Q3 Who likes Berta?

A3 Berta

Questions about function-valued functions

$$[\![\textbf{likes}]\!] = \begin{bmatrix} \text{Anna} & \rightarrow & \begin{bmatrix} \text{Anna} & \rightarrow & 1 \\ \text{Berta} & \rightarrow & 1 \\ \text{Clara} & \rightarrow & 1 \end{bmatrix} \\ \text{Berta} & \rightarrow & \begin{bmatrix} \text{Anna} & \rightarrow & 0 \\ \text{Berta} & \rightarrow & 1 \\ \text{Clara} & \rightarrow & 0 \end{bmatrix} \\ \text{Clara} & \rightarrow & \begin{bmatrix} \text{Anna} & \rightarrow & 1 \\ \text{Berta} & \rightarrow & 1 \\ \text{Clara} & \rightarrow & 0 \end{bmatrix} \end{bmatrix}$$

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Questions about function-valued functions

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Questions about function-valued functions

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Questions about function-valued functions

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Q4 What is the value of $[\![\textbf{likes}]\!]$ applied to Clara?

A4 $[\![\textbf{likes}]\!](\text{Clara}) = \begin{bmatrix} \text{Anna} & \to & 1 \\ \text{Berta} & \to & 1 \\ \text{Clara} & \to & 0 \end{bmatrix}$

Questions about function-valued functions

$$[\![\textbf{likes}]\!] = \left[\begin{array}{lll} \text{Anna} & \rightarrow & \left[\begin{array}{lll} \text{Anna} & \rightarrow & 1 \\ \text{Berta} & \rightarrow & 1 \\ \text{Clara} & \rightarrow & 1 \end{array}\right] \\ \text{Berta} & \rightarrow & \left[\begin{array}{lll} \text{Anna} & \rightarrow & 0 \\ \text{Berta} & \rightarrow & 1 \\ \text{Clara} & \rightarrow & 0 \end{array}\right] \\ \text{Clara} & \rightarrow & \left[\begin{array}{lll} \text{Anna} & \rightarrow & 1 \\ \text{Berta} & \rightarrow & 1 \\ \text{Clara} & \rightarrow & 0 \end{array}\right] \end{array}\right]$$

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A4 $[\![\textbf{likes}]\!](\text{Clara}) = \left[\begin{array}{lll} \text{Anna} & \rightarrow & 1 \\ \text{Berta} & \rightarrow & 1 \\ \text{Clara} & \rightarrow & 0 \end{array}\right]$

Questions about function-valued functions

$$[\![\textbf{likes}]\!] = \begin{bmatrix} \text{Anna} & \rightarrow & \begin{bmatrix} \text{Anna} & \rightarrow & 1 \\ \text{Berta} & \rightarrow & 1 \\ \text{Clara} & \rightarrow & 1 \end{bmatrix} \\ \text{Berta} & \rightarrow & \begin{bmatrix} \text{Anna} & \rightarrow & 0 \\ \text{Berta} & \rightarrow & 1 \\ \text{Clara} & \rightarrow & 0 \end{bmatrix} \\ \text{Clara} & \rightarrow & \begin{bmatrix} \text{Anna} & \rightarrow & 1 \\ \text{Berta} & \rightarrow & 1 \\ \text{Clara} & \rightarrow & 0 \end{bmatrix} \end{bmatrix}$$

Q5 What is the value if the value of $[\![\textbf{likes}]\!]$ applied to Berta is applied to Clara?

A5 $[\![\textbf{likes}]\!](\text{Berta})(\text{Clara}) = \begin{bmatrix} \text{Anna} & \rightarrow & 0 \\ \text{Berta} & \rightarrow & 1 \\ \text{Clara} & \rightarrow & 0 \end{bmatrix}(\text{Clara}) = 0$

Questions about function-valued functions

$$[\![\textbf{likes}]\!] = \left[\begin{array}{lcl} \text{Anna} & \to & \left[\begin{array}{lcl} \text{Anna} & \to & 1 \\ \text{Berta} & \to & 1 \\ \text{Clara} & \to & 1 \end{array}\right] \\ \text{Berta} & \to & \left[\begin{array}{lcl} \text{Anna} & \to & 0 \\ \text{Berta} & \to & 1 \\ \text{Clara} & \to & 0 \end{array}\right] \\ \text{Clara} & \to & \left[\begin{array}{lcl} \text{Anna} & \to & 1 \\ \text{Berta} & \to & 1 \\ \text{Clara} & \to & 0 \end{array}\right] \end{array}\right]$$

Q5 What is the value if the value of ⟦**likes**⟧ applied to Berta is applied to Clara?

A5 $[\![\textbf{likes}]\!](\text{Berta})(\text{Clara}) = \left[\begin{array}{lcl} \text{Anna} & \to & 0 \\ \text{Berta} & \to & 1 \\ \text{Clara} & \to & 0 \end{array}\right](\text{Clara}) = 0$

Questions about function-valued functions

$$[\![\textbf{likes}]\!] = \begin{bmatrix} \text{Anna} & \to & \begin{bmatrix} \text{Anna} & \to & 1 \\ \text{Berta} & \to & 1 \\ \text{Clara} & \to & 1 \end{bmatrix} \\ \text{Berta} & \to & \begin{bmatrix} \text{Anna} & \to & 0 \\ \text{Berta} & \to & 1 \\ \text{Clara} & \to & 0 \end{bmatrix} \\ \text{Clara} & \to & \begin{bmatrix} \text{Anna} & \to & 1 \\ \text{Berta} & \to & 1 \\ \text{Clara} & \to & 0 \end{bmatrix} \end{bmatrix}$$

Q6 Which information is in $[\![\textbf{likes}]\!]$(Anna)(Clara) $= 1$?

A6 The information that Clara likes Anna

Q7 Which information is in $[\![\textbf{likes}]\!]$(Clara)?

A7 The information who likes Clara and who does not.

Questions about function-valued functions

$$[\![\textbf{likes}]\!] = \left[\begin{array}{lcl} \text{Anna} & \to & \left[\begin{array}{lcl} \text{Anna} & \to & 1 \\ \text{Berta} & \to & 1 \\ \text{Clara} & \to & 1 \end{array}\right] \\ \text{Berta} & \to & \left[\begin{array}{lcl} \text{Anna} & \to & 0 \\ \text{Berta} & \to & 1 \\ \text{Clara} & \to & 0 \end{array}\right] \\ \text{Clara} & \to & \left[\begin{array}{lcl} \text{Anna} & \to & 1 \\ \text{Berta} & \to & 1 \\ \text{Clara} & \to & 0 \end{array}\right] \end{array}\right]$$

Q6 Which information is in $[\![\textbf{likes}]\!](\text{Anna})(\text{Clara}) = 1$?

A6 The information that Clara likes Anna

Q7 Which information is in $[\![\textbf{likes}]\!](\text{Clara})$?

A7 The information who likes Clara and who does not.

Questions about function-valued functions

$$[\![\textbf{likes}]\!] = \begin{bmatrix} \text{Anna} & \rightarrow & \begin{bmatrix} \text{Anna} & \rightarrow & 1 \\ \text{Berta} & \rightarrow & 1 \\ \text{Clara} & \rightarrow & 1 \end{bmatrix} \\ \text{Berta} & \rightarrow & \begin{bmatrix} \text{Anna} & \rightarrow & 0 \\ \text{Berta} & \rightarrow & 1 \\ \text{Clara} & \rightarrow & 0 \end{bmatrix} \\ \text{Clara} & \rightarrow & \begin{bmatrix} \text{Anna} & \rightarrow & 1 \\ \text{Berta} & \rightarrow & 1 \\ \text{Clara} & \rightarrow & 0 \end{bmatrix} \end{bmatrix}$$

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Q7 Which information is in $[\![\textbf{likes}]\!](\text{Clara})$?

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Questions about function-valued functions

$$[\![\textbf{likes}]\!] = \begin{bmatrix} \text{Anna} & \rightarrow & \begin{bmatrix} \text{Anna} & \rightarrow & 1 \\ \text{Berta} & \rightarrow & 1 \\ \text{Clara} & \rightarrow & 1 \end{bmatrix} \\ \text{Berta} & \rightarrow & \begin{bmatrix} \text{Anna} & \rightarrow & 0 \\ \text{Berta} & \rightarrow & 1 \\ \text{Clara} & \rightarrow & 0 \end{bmatrix} \\ \text{Clara} & \rightarrow & \begin{bmatrix} \text{Anna} & \rightarrow & 1 \\ \text{Berta} & \rightarrow & 1 \\ \text{Clara} & \rightarrow & 0 \end{bmatrix} \end{bmatrix}$$

Q6 Which information is in $[\![\textbf{likes}]\!](\text{Anna})(\text{Clara}) = 1$?

A6 The information that Clara likes Anna

Q7 Which information is in $[\![\textbf{likes}]\!](\text{Clara})$?

A7 The information who likes Clara and who does not.

New inventory of denotations

- Elements of D: the set of actual individuals
- Elements of $\{0, 1\}$: the set of truth-values
- Functions from D to $\{0, 1\}$
- Functions from $\{0, 1\}$ to $\{0, 1\}$
- Functions from D to functions from D to $\{0, 1\}$

Lexical entries

- Proper names

  $[\![\textbf{Mary}]\!]$ = Mary
  $[\![\textbf{Bill}]\!]$ = Bill

- Intransitive verbs

  $[\![\textbf{dances}]\!] = f : D \rightarrow \{0, 1\}$
  For all $x \in D$, $f(x) = 1$ iff x dances
  $[\![\textbf{smokes}]\!] = f : D \rightarrow \{0, 1\}$
  For all $x \in D$, $f(x) = 1$ iff x smokes

- Operators, connectives

  $[\![\textbf{not}]\!] = f : \{0, 1\} \rightarrow \{0, 1\}$
  For all $x \in \{0, 1\}$, $f(x) = 1$ iff $x = 0$

- Transitive verbs

  $[\![\textbf{likes}]\!] = f : D \rightarrow \{g : g \text{ is a function from } D \text{ to } \{0, 1\}\}$
  For all $x, y \in D$, $f(x)(y) = 1$ iff y likes x
  $[\![\textbf{hits}]\!] = f : D \rightarrow \{g : g \text{ is a function from } D \text{ to } \{0, 1\}\}$
  For all $x, y \in D$, $f(x)(y) = 1$ iff y hits x

Lexical entries

- Proper names

  $[\![\textbf{Mary}]\!] = \text{Mary}$
  $[\![\textbf{Bill}]\!] = \text{Bill}$

- Intransitive verbs

  $[\![\textbf{dances}]\!] = f : D \rightarrow \{0, 1\}$
  For all $x \in D$, $f(x) = 1$ iff x dances
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  For all $x \in D$, $f(x) = 1$ iff x smokes

- Operators, connectives

  $[\![\textbf{not}]\!] = f : \{0, 1\} \rightarrow \{0, 1\}$
  For all $x \in \{0, 1\}$, $f(x) = 1$ iff $x = 0$

- Transitive verbs

  $[\![\textbf{likes}]\!] = f : D \rightarrow \{g : g \text{ is a function from } D \text{ to } \{0, 1\}\}$
  For all $x, y \in D$, $f(x)(y) = 1$ iff y likes x
  $[\![\textbf{hits}]\!] = f : D \rightarrow \{g : g \text{ is a function from } D \text{ to } \{0, 1\}\}$
  For all $x, y \in D$, $f(x)(y) = 1$ iff y hits x

Defining function-valued functions I

We are using a simplified definition

$$[\![\textbf{likes}]\!] = \begin{array}{l} f : D \rightarrow \{g : g \text{ is a function from } D \text{ to } \{0,1\}\} \\ \text{For all } x, y \in D, f(x)(y) = 1 \text{ iff } y \text{ likes } x \end{array}$$

Actually, there is another function g_x embedded in function f.

g_x is the value of $f(x)$.

The value is a function ('function-valued function')

$$[\![\textbf{likes}]\!] = \begin{array}{l} f : D \rightarrow \{g : g \text{ is a function from } D \text{ to } \{0,1\}\} \\ \text{For all } x \in D, f(x) = \begin{array}{l} g_x : D \rightarrow \{0,1\} \\ \text{For all } y \in D, g_x(y) = 1 \text{ iff } y \text{ likes } x. \end{array} \end{array}$$

Defining function-valued functions I

We are using a simplified definition

$$[\![\textbf{likes}]\!] = f : D \rightarrow \{g : g \text{ is a function from } D \text{ to } \{0,1\}\}$$
$$\text{For all } x, y \in D, f(x)(y) = 1 \text{ iff } y \text{ likes } x$$

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$$[\![\textbf{likes}]\!] = \begin{array}{l} f : D \rightarrow \{g : g \text{ is a function from } D \text{ to } \{0,1\}\} \\ \text{For all } x \in D, f(x) = \begin{array}{l} g_x : D \rightarrow \{0,1\} \\ \text{For all } y \in D, g_x(y) = 1 \text{ iff } y \text{ likes } x. \end{array} \end{array}$$

Defining function-valued functions II

$$[\![\textbf{likes}]\!] = \begin{array}{l} f : D \to \{g : g \text{ is a function from } D \text{ to } \{0,1\}\} \\ \text{For all } x \in D, f(x) = \begin{array}{l} g_x : D \to \{0,1\} \\ \text{For all } y \in D, g_x(y) = 1 \text{ iff } y \text{ likes } x. \end{array} \end{array}$$

'the function f from D to the set of functions from D to $\{0, 1\}$ such that for every $x \in D$, $f(x)$ is the function g_x from D to $\{0, 1\}$ such that for all $y \in D$, $g_x(y) = 1$ iff y likes x'

$[\![\textbf{likes}]\!]$ is a unary function (takes exactly one argument)

Arguments of $[\![\textbf{likes}]\!]$ are interpreted as the liked ones.

Follows from phrase-structure.

Semantic rules

S1 If α has the form $[_{S}\ \beta\ \gamma]$, then $[\![\alpha]\!] = [\![\gamma]\!]([\![\beta]\!])$.

S2 If α has the form $[_{NP}\ \beta]$, then $[\![\alpha]\!] = [\![\beta]\!]$.

S3 If α has the form $[_{VP}\ \beta]$, then $[\![\alpha]\!] = [\![\beta]\!]$.

S4 If α has the form $[_{N}\ \beta]$, then $[\![\alpha]\!] = [\![\beta]\!]$.

S5 If α has the form $[_{V}\ \beta]$, then $[\![\alpha]\!] = [\![\beta]\!]$.

S6 If α has the form $[_{S}\ \textbf{not}\ \beta]$, then $[\![\alpha]\!] = [\![\textbf{not}]\!]([\![\beta]\!])$.

S7 If α has the form $[_{VP}\ \beta\ \gamma]$, then $[\![\alpha]\!] = [\![\beta]\!]([\![\gamma]\!])$.

Semantic rules

S1 If α has the form $[_{S}\ \beta\ \gamma]$, then $[\![\alpha]\!] = [\![\gamma]\!]([\![\beta]\!])$.

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S7 If α has the form $[_{VP}\ \beta\ \gamma]$, then $[\![\alpha]\!] = [\![\beta]\!]([\![\gamma]\!])$.

Truth-conditions

$[\![\mathbf{S}]\!] = 1$ iff Mary likes Bill (assumption)

$[\![\mathbf{S}]\!] = [\![\mathbf{VP}]\!]([\![\mathbf{NP'}]\!])$ (S1)

$= [\![\mathbf{V}]\!]([\![\mathbf{NP}]\!])([\![\mathbf{NP'}]\!])$ (S7)

$= [\![\mathbf{V}]\!]([\![\mathbf{N}]\!])([\![\mathbf{N'}]\!])$ (2×S2)

$= [\![\mathbf{V}]\!]([\![\mathbf{Bill}]\!])([\![\mathbf{Mary}]\!])$ (2×S4)

$= [\![\mathbf{likes}]\!]([\![\mathbf{Bill}]\!])([\![\mathbf{Mary}]\!])$ (S5)

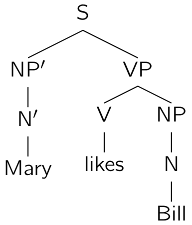
$= \left[\begin{array}{l} f : D \to \{g : g \text{ is a function from } D \text{ to } \{0,1\}\} \\ \text{For all } x, y \in D, f(x)(y) = 1 \text{ iff } y \text{ likes } x \end{array} \right]$

(Bill)(Mary) (3×lexicon)

$\left[\begin{array}{l} f : D \to \{g : g \text{ is a function from } D \text{ to } \{0,1\}\} \\ \text{For all } x, y \in D, f(x)(y) = 1 \text{ iff } y \text{ likes } x \end{array} \right]$

(Bill)(Mary) = 1 iff Mary likes Bill

$[\![\mathbf{S}]\!] = 1$ iff Mary likes Bill (QED.)



Truth-conditions

$$[\![\mathbf{S}]\!] = 1 \text{ iff Mary likes Bill} \qquad \text{(assumption)}$$

$$[\![\mathbf{S}]\!] = [\![\mathbf{VP}]\!]([\![\mathbf{NP'}]\!]) \qquad \text{(S1)}$$

$$= [\![\mathbf{V}]\!]([\![\mathbf{NP}]\!])([\![\mathbf{NP'}]\!]) \qquad \text{(S7)}$$

$$= [\![\mathbf{V}]\!]([\![\mathbf{N}]\!])([\![\mathbf{N'}]\!]) \qquad (2\times\text{S2})$$

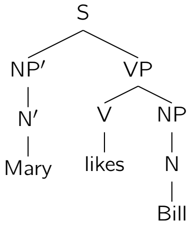
$$= [\![\mathbf{V}]\!]([\![\mathbf{Bill}]\!])([\![\mathbf{Mary}]\!]) \qquad (2\times\text{S4})$$

$$= [\![\mathbf{likes}]\!]([\![\mathbf{Bill}]\!])([\![\mathbf{Mary}]\!]) \qquad \text{(S5)}$$

$$= \left[\begin{array}{l} f : D \to \{g : g \text{ is a function from } D \text{ to } \{0,1\}\} \\ \text{For all } x, y \in D, f(x)(y) = 1 \text{ iff } y \text{ likes } x \end{array}\right] (\text{Bill})(\text{Mary}) \qquad (3\times\text{lexicon})$$

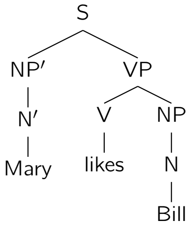
$$\left[\begin{array}{l} f : D \to \{g : g \text{ is a function from } D \text{ to } \{0,1\}\} \\ \text{For all } x, y \in D, f(x)(y) = 1 \text{ iff } y \text{ likes } x \end{array}\right] (\text{Bill})(\text{Mary}) = 1 \text{ iff Mary likes Bill}$$

$$[\![\mathbf{S}]\!] = 1 \text{ iff Mary likes Bill} \qquad \text{(QED.)}$$



Truth-conditions

$$
\begin{aligned}
[\![\mathbf{S}]\!] &= 1 \text{ iff Mary likes Bill} && \text{(assumption)}\\
[\![\mathbf{S}]\!] &= [\![\mathbf{VP}]\!]([\![\mathbf{NP'}]\!]) && \text{(S1)}\\
&= [\![\mathbf{V}]\!]([\![\mathbf{NP}]\!])([\![\mathbf{NP'}]\!]) && \text{(S7)}\\
&= [\![\mathbf{V}]\!]([\![\mathbf{N}]\!])([\![\mathbf{N'}]\!]) && (2\times\text{S2})\\
&= [\![\mathbf{V}]\!]([\![\mathbf{Bill}]\!])([\![\mathbf{Mary}]\!]) && (2\times\text{S4})\\
&= [\![\mathbf{likes}]\!]([\![\mathbf{Bill}]\!])([\![\mathbf{Mary}]\!]) && \text{(S5)}\\
&= \left[\begin{array}{l} f : D \rightarrow \{g : g \text{ is a function from } D \text{ to } \{0,1\}\} \\ \text{For all } x, y \in D, f(x)(y) = 1 \text{ iff } y \text{ likes } x \end{array}\right]\\
&\quad \text{(Bill)(Mary)} && (3\times\text{lexicon})\\
&\left[\begin{array}{l} f : D \rightarrow \{g : g \text{ is a function from } D \text{ to } \{0,1\}\} \\ \text{For all } x, y \in D, f(x)(y) = 1 \text{ iff } y \text{ likes } x \end{array}\right]\\
&\text{(Bill)(Mary)} = 1 \text{ iff Mary likes Bill}\\
[\![\mathbf{S}]\!] &= 1 \text{ iff Mary likes Bill} && \text{(QED.)}
\end{aligned}
$$



Truth-conditions

$$
\begin{aligned}
[\![\mathbf{S}]\!] &= 1 \text{ iff Mary likes Bill} && \text{(assumption)}\\
[\![\mathbf{S}]\!] &= [\![\mathbf{VP}]\!]([\![\mathbf{NP'}]\!]) && \text{(S1)}\\
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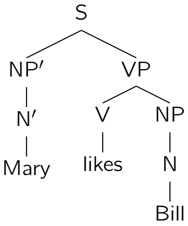
&= [\![\mathbf{V}]\!]([\![\mathbf{N}]\!])([\![\mathbf{N'}]\!]) && (2\times\text{S2})\\
&= [\![\mathbf{V}]\!]([\![\mathbf{Bill}]\!])([\![\mathbf{Mary}]\!]) && (2\times\text{S4})\\
&= [\![\mathbf{likes}]\!]([\![\mathbf{Bill}]\!])([\![\mathbf{Mary}]\!]) && \text{(S5)}\\
&= \begin{bmatrix} f : D \to \{g : g \text{ is a function from } D \text{ to } \{0,1\}\} \\ \text{For all } x, y \in D, f(x)(y) = 1 \text{ iff } y \text{ likes } x \end{bmatrix}\\
&\quad \text{(Bill)(Mary)} && (3\times\text{lexicon})\\
&\begin{bmatrix} f : D \to \{g : g \text{ is a function from } D \text{ to } \{0,1\}\} \\ \text{For all } x, y \in D, f(x)(y) = 1 \text{ iff } y \text{ likes } x \end{bmatrix}\\
&\text{(Bill)(Mary)} = 1 \text{ iff Mary likes Bill}\\
[\![\mathbf{S}]\!] &= 1 \text{ iff Mary likes Bill} && \text{(QED.)}
\end{aligned}
$$



Truth-conditions

$$[\![\textbf{S}]\!] = 1 \text{ iff Mary likes Bill} \qquad \text{(assumption)}$$

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$$= [\![\textbf{V}]\!]([\![\textbf{N}]\!])([\![\textbf{N}']\!]) \qquad (2\times\text{S2})$$

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$$= [\![\textbf{likes}]\!]([\![\textbf{Bill}]\!])([\![\textbf{Mary}]\!]) \qquad \text{(S5)}$$

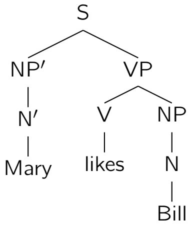
$$= \begin{bmatrix} f : D \to \{g : g \text{ is a function from } D \text{ to } \{0,1\}\} \\ \text{For all } x, y \in D, f(x)(y) = 1 \text{ iff } y \text{ likes } x \end{bmatrix}$$

$$\text{(Bill)(Mary)} \qquad (3\times\text{lexicon})$$

$$\begin{bmatrix} f : D \to \{g : g \text{ is a function from } D \text{ to } \{0,1\}\} \\ \text{For all } x, y \in D, f(x)(y) = 1 \text{ iff } y \text{ likes } x \end{bmatrix}$$

$$\text{(Bill)(Mary)} = 1 \text{ iff Mary likes Bill}$$

$$[\![\textbf{S}]\!] = 1 \text{ iff Mary likes Bill} \qquad \text{(QED.)}$$



Truth-conditions

$$
\begin{aligned}
[\![\mathbf{S}]\!] &= 1 \text{ iff Mary likes Bill} && \text{(assumption)}\\
[\![\mathbf{S}]\!] &= [\![\mathbf{VP}]\!]([\![\mathbf{NP'}]\!]) && \text{(S1)}\\
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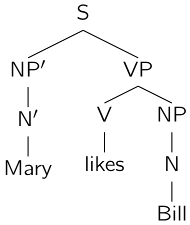
&= [\![\mathbf{V}]\!]([\![\mathbf{N}]\!])([\![\mathbf{N'}]\!]) && (2\times\text{S2})\\
&= [\![\mathbf{V}]\!]([\![\mathbf{Bill}]\!])([\![\mathbf{Mary}]\!]) && (2\times\text{S4})\\
&= [\![\mathbf{likes}]\!]([\![\mathbf{Bill}]\!])([\![\mathbf{Mary}]\!]) && \text{(S5)}\\
&= \left[\begin{array}{l} f : D \to \{g : g \text{ is a function from } D \text{ to } \{0,1\}\} \\ \text{For all } x, y \in D, f(x)(y) = 1 \text{ iff } y \text{ likes } x \end{array}\right] \\
&\quad (\text{Bill})(\text{Mary}) && (3\times\text{lexicon})\\
&\left[\begin{array}{l} f : D \to \{g : g \text{ is a function from } D \text{ to } \{0,1\}\} \\ \text{For all } x, y \in D, f(x)(y) = 1 \text{ iff } y \text{ likes } x \end{array}\right] \\
&(\text{Bill})(\text{Mary}) = 1 \text{ iff Mary likes Bill}\\
[\![\mathbf{S}]\!] &= 1 \text{ iff Mary likes Bill} && \text{(QED.)}
\end{aligned}
$$



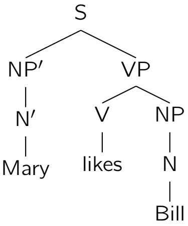
Truth-conditions

$$
\begin{aligned}
\llbracket \textbf{S} \rrbracket &= 1 \text{ iff Mary likes Bill} && \text{(assumption)} \\
\llbracket \textbf{S} \rrbracket &= \llbracket \textbf{VP} \rrbracket(\llbracket \textbf{NP}' \rrbracket) && \text{(S1)} \\
&= \llbracket \textbf{V} \rrbracket(\llbracket \textbf{NP} \rrbracket)(\llbracket \textbf{NP}' \rrbracket) && \text{(S7)} \\
&= \llbracket \textbf{V} \rrbracket(\llbracket \textbf{N} \rrbracket)(\llbracket \textbf{N}' \rrbracket) && (2{\times}\text{S2}) \\
&= \llbracket \textbf{V} \rrbracket(\llbracket \textbf{Bill} \rrbracket)(\llbracket \textbf{Mary} \rrbracket) && (2{\times}\text{S4}) \\
&= \llbracket \textbf{likes} \rrbracket(\llbracket \textbf{Bill} \rrbracket)(\llbracket \textbf{Mary} \rrbracket) && \text{(S5)} \\
&= \left[\begin{array}{l} f : D \to \{g : g \text{ is a function from } D \text{ to } \{0,1\}\} \\ \text{For all } x, y \in D,\ f(x)(y) = 1 \text{ iff } y \text{ likes } x \end{array} \right] \\
&\quad \text{(Bill)(Mary)} && (3{\times}\text{lexicon})
\end{aligned}
$$

$$
\left[\begin{array}{l} f : D \to \{g : g \text{ is a function from } D \text{ to } \{0,1\}\} \\ \text{For all } x, y \in D,\ f(x)(y) = {\color{red}1 \text{ iff } y \text{ likes } x} \end{array} \right]
$$

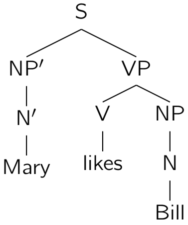
$$
\text{(Bill)(Mary)} = {\color{red}1 \text{ iff Mary likes Bill}}
$$

$$
\llbracket \textbf{S} \rrbracket = 1 \text{ iff Mary likes Bill} \qquad \text{(QED.)}
$$



Truth-conditions

$$
\begin{aligned}
[\![\mathbf{S}]\!] &= 1 \text{ iff Mary likes Bill} && \text{(assumption)}\\
[\![\mathbf{S}]\!] &= [\![\mathbf{VP}]\!]([\![\mathbf{NP'}]\!]) && \text{(S1)}\\
&= [\![\mathbf{V}]\!]([\![\mathbf{NP}]\!])([\![\mathbf{NP'}]\!]) && \text{(S7)}\\
&= [\![\mathbf{V}]\!]([\![\mathbf{N}]\!])([\![\mathbf{N'}]\!]) && (2\times\text{S2})\\
&= [\![\mathbf{V}]\!]([\![\mathbf{Bill}]\!])([\![\mathbf{Mary}]\!]) && (2\times\text{S4})\\
&= [\![\mathbf{likes}]\!]([\![\mathbf{Bill}]\!])([\![\mathbf{Mary}]\!]) && \text{(S5)}\\
&= \left[\begin{array}{l} f : D \to \{g : g \text{ is a function from } D \text{ to } \{0,1\}\} \\ \text{For all } x, y \in D, f(x)(y) = 1 \text{ iff } y \text{ likes } x \end{array}\right] \\
&\quad \text{(Bill)(Mary)} && (3\times\text{lexicon})\\
&\left[\begin{array}{l} f : D \to \{g : g \text{ is a function from } D \text{ to } \{0,1\}\} \\ \text{For all } x, y \in D, f(x)(y) = 1 \text{ iff } y \text{ likes } x \end{array}\right] \\
&\text{(Bill)(Mary)} = 1 \text{ iff Mary likes Bill} \\
[\![\mathbf{S}]\!] &= 1 \text{ iff Mary likes Bill} && \text{(QED.)}
\end{aligned}
$$



Overview

1. What we did yesterday
2. How to expand the system
3. Sets and their characteristic functions
4. Transitive verbs
5. Semantic types

Categorizing denotation types

The set of denotation types has become complex.

They can be more straightforwardly expressed.

Semantic types and denotation domains

Semantic types

- e and t are semantic types
- If σ and τ are semantic types, $\langle \sigma, \tau \rangle$ is a semantic type.
- Nothing else is a semantic type.

- e ('entity') stands for individual
- t ('truth-value') stands for truth-value
- $\langle \sigma, \tau \rangle$ stands for 'function from type σ to type τ'

Semantic denotation domains

- $D_e := D$ (set of individuals)
- $D_t := \{0, 1\}$ (set of truth-values)
- For all semantic types σ and τ, $D_{\langle \sigma, \tau \rangle}$ is the set of all function from D_σ to D_τ.

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Inventory of denotation types

- Elements of D_e, where $D_e := D$
- Elements of D_t, where $D_t := \{0, 1\}$
- Elements of $D_{\langle e,t \rangle}$, where $D_{\langle e,t \rangle} := \{f : f$ is a function from D_e to $D_t\}$
- Elements of $D_{\langle e,\langle e,t \rangle\rangle}$, where $D_{\langle e,\langle e,t \rangle\rangle} := \{f : f$ is a function from D_e to $D_{\langle e,t \rangle}\}$
- Elements of $D_{\langle t,t \rangle}$, wo $D_{\langle t,t \rangle} := \{f : f$ is a function from D_t to $D_t\}$

- $\langle e, t \rangle$ stands for 'function from D_e to D_t'
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Questions about semantic types

Q1 What is the semantic type of ⟦**dances**⟧?

A1 $\langle e, t \rangle$

Q2 What is the semantic type of ⟦**likes**⟧?

A2 $\langle e, \langle e, t \rangle \rangle$

Q3 What is the semantic type of ⟦**John**⟧?

A3 e

Q4 What is the semantic type of ⟦**likes**⟧(⟦**John**⟧)?

A4 $\langle e, t \rangle$

Q5 What is the semantic type of ⟦**likes**⟧(⟦**John**⟧)(⟦**Mary**⟧)?

A5 t

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Q1 What is the semantic type of ⟦**dances**⟧?

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A3 e

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A4 $\langle e, t \rangle$

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Q3 What is the semantic type of $[\![$**John**$]\!]$?

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Q4 What is the semantic type of $[\![$**likes**$]\!]([\![$**John**$]\!])$?

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Q1 $D_e \subseteq D_t$: true?

A1 No, no $x \in D_e$ is in D_t, because no $x \in D_e$ is a truth-value

Q2 $D_e \in D_{\langle e,t \rangle}$

A3 No. $D_{\langle e,t \rangle}$ is the set of functions from D_e to $\{0, 1\}$. D_e is the set of individuals D and therefore not a function from D_e to $\{0, 1\}$.

Q3 $[\![\textbf{likes}]\!] \in D_{\langle e,\langle e,t \rangle\rangle}$

A3 Yes. $[\![\textbf{likes}]\!]$ is a function from D_e to functions from D_e to D_t. Thus $[\![\textbf{likes}]\!]$ is a member of the set of functions from D_e to functions from D_e to D_t.

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