Introduction to semantics

Heim & Kratzer 1998, chapters 1-2

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July 25, 2016

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Overview

1. Tasks of a semantic theory
2. Sets
3. Functions
4. How syntax determines sentence interpretation
5. The three components of compositional interpretation

Two questions about the meaning of sentences

(1) **Tony swims.**

1. How should we characterize the meaning of (1)?

2. How does this meaning come about?

Intuitive answer to the two questions

The meaning of (1) is straightforward: Tony swims

(1) **Tony swims.**

- **Tony** = the person named Tony
- **swims** = carry out the action of swimming
- **Tony swims** = the person named Tony carries out the action of swimming

Do we need to say more?

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Do we need to say more?

Making question 1 clearer

When we ask,

"How should we characterize the meaning of a sentence S?"

what we as linguists really mean is

"What kind of semantic competence allows a native speaker to understand S?"

Truth and falsity

Situation: Your friend Tony is in the swimming pool. He moves his arms and feet in such a way as to avoid drowning.

✓ **Tony swims.**

Situation: Your friend Tony is in the swimming pool. He is standing in the shallow end of the pool drinking a beer.

↯ **Tony swims.**

Semantic competence 1

The semantic competence of a native speaker among other things includes:

To know when a sentence S is true and when S is false.

We want to understand what enables a speaker to know this.

Semantic competence 2

A speaker knows that when (1) is true, (2) is true as well, but not necessarily vice versa.

(1) **Tony swims.**

(2) **Someone swims.**

A speaker knows which sentences entail other sentences.

Literal meaning

Literal meaning of S = truth-conditions of S (Alfred Tarski)

Truth-conditions of S in a particular situation s yield the truth-value of S

Truth-conditions

Tony swims. = true in situation *s* iff Tony swims

This is not a trivial statement. Languages differ in the expressions they use, but the truth-conditions remain the same.

Tony plavaet = true in situation *s* iff Tony swims

Truth-conditions

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Truth-conditions and entailment

S entails S' iff whenever S is true S' is true

(1) entails ($\Rightarrow$) (2)

(1) **Tony swims.**

(2) **Someone swims.**

test: contradiction if (2) is false and (1) true.

The truth-conditions for (1) and (2) must guarantee this entailment relation.

Making question 2 clearer

When we ask

"How does this meaning—i.e., the truth-conditions—come about?"

what we as linguists really mean is

"What processes guarantee the truth-conditions that we observe?"

How come **Tony swims** is true iff Tony swims, not, say, iff Tony barks?

Frege's principle of compositionality

The truth-conditions of a sentence S are a function of the meaning of S's parts.

Semantic composition consists in function application.

A semantic theory gives us . . .

The truth-conditions of every possible sentence in a language.

A compositional process determining these truth-conditions.

In order to achieve this, one needs to know at least

1. the meanings of lexical items
2. the compositional rules assigning meanings to complex expressions

A semantic theory does not give us ...

Non-literal meaning components such as implicatures.

(3) suggests that not everyone swims.

(3) **Some of the boys swim.**

This inference is cancelable by continuing with **In fact, all do**.

Given the cancelability, the inference cannot be part of the truth-conditions.

Its derivation is the job of a pragmatic theory.

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Sets

A set is an unordered collection of objects.

These objects are the members/elements of the set.

- $\mathbb{N}$ is the set of natural numbers
- $2 \in \mathbb{N}$ (2 is a member/element of $\mathbb{N}$)
- $1/2 \notin \mathbb{N}$ ($1/2$ is not a member/element of $\mathbb{N}$)

Set without members is the empty set, $\varnothing$.

The members are enclosed by curly brackets: e.g. $\{a, b, c\}$

Why sets?

Semantic composition and therefore truth-conditions are characterizable by sets and set-membership, which are in turn characterized by functions and function application, respectively.

swim = set of swimmers

Tony = Tony

Tony swims is true iff Tony is a member of the set of swimmers

Set equivalence

Sets A and B are equivalent iff A and B have exactly the same elements.

$$\{a, b, c\} = \{a, b, c\}$$

Since sets are unordered

$$\{a, b, c\} = \{b, c, a\} = \{b, a, c\} = \{c, a, b\} = \{c, b, a\} \ldots$$

Subset

A is a subset of B iff all members of A are members of B.

$$\{a, b, c\} \subseteq \{a, b, c, d\}$$

Each set is a subset of itself.

$$\{a, b, c\} \subseteq \{a, b, c\}$$

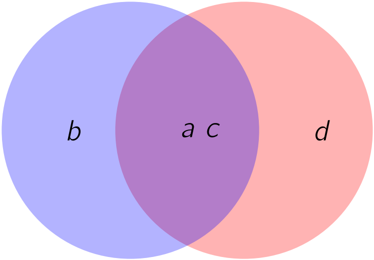
A is a proper subset of B iff A is a subset of B but not equivalent to B

$$\{a, b, c\} \subset \{a, b, c, d\}$$

Intersection

The intersection A and B, $A \cap B$, is the C with exactly those elements which are shared by A and B.

$$\{a, b, c\} \cap \{a, c, d\} = \{a, c\}$$



Overlap and disjointness

A and B overlap iff $A \cap B \neq \varnothing$.

$$\{a, b, c, d\} \neq \{d, e, f\}$$

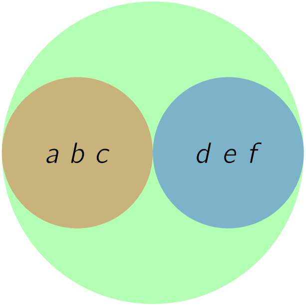
A and B are disjoint iff $A \cap B = \varnothing$.

$$\{a, b, c\} \neq \{d, e, f\}$$

Union

The union of A and B, $A \cup B$, is the C with all the members of A and B and nothing else.

$$\{a, b, c\} \cup \{d, e, f\} = \{a, b, c, d, e, f\}$$



Complement

The complement of A in B, $B - A$, is the C with all elements of B which are not in A and nothing else.

$$\{a, b, c, d\} - \{a, b\} = \{c, d\}$$

$$\{a, b, c\} - \{a, b, d\} = \{c\}$$

Defining sets

$A := \ldots$

'Let A be that set, ...'

Defining sets via listing

$A := \{a, b, c\}$

'Let A be that set, whose elements are a, b, c, and nothing else.'

Abstraction

Sets can be defined via abstraction so that not all members have to be listed, which is helpful when there is an infinite number of members or the members are not known.

$$A := \{x : x \text{ is a president}\}$$

x is a variable

All elements in A must satisfy requirement to the right of :

'Let A be that set such that for all $x \in A$ it holds that x is a president'

'Let A be the set of presidents'

Set membership

To determine whether object a is a member of A, the variable is set to a.

If the result is true, $a \in A$

If the result is false, $a \notin A$.

$A := \{x : x \text{ is a president}\}$

Obama is a president is true
Obama $\in A$

Sarkozy is a president is false
Sarkozy $\notin A$

Notation

- x, y, z are variables ranging over objects
- a, b, c are names of objects
- X, Y, Z are variables ranging over sets
- A, B, C are names of sets

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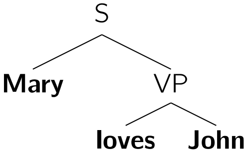
Why functions?

Semantic composition and therefore truth-conditions are characterizable by sets and set-membership, which are in turn characterized by functions and function application, respectively. Functions are types of sets.

The meaning of **loves** is applied to the meaning of **John**. The result, the meaning of **VP** is applied to the meaning of **Mary**.



Ordered pairs

Pairs of objects are ordered

a in $\langle a, b \rangle$: left component of the pair $\langle a, b \rangle$

b in $\langle a, b \rangle$: right component of the pair $\langle a, b \rangle$

$$\langle a, b \rangle \neq \langle b, a \rangle \text{ (except for when } a = b\text{)}$$

Relations

A (binary) relation is a set of ordered pairs.

$$\{\langle a, b\rangle, \langle c, d\rangle\}$$

Relations are defined by two conditions on two variables to the right of :

$$R := \{\langle x, y\rangle : x \text{ is an author and } y \text{ is a book of } x\}$$

$\langle$Thomas Mann, Buddenbrooks$\rangle \in R$
$\langle$Leo Tolstoy, War & Peace$\rangle \in R$

Functions

Functions are relations satisfying the uniqueness condition: each left component of an ordered pair in it has exactly one right component.

A relation f is a function iff for every x, such that there are y and z such that $\langle x, y \rangle \in f$ and $\langle x, z \rangle \in f$, $y = z$.

Domain and range I

Each function has a domain and a range.

The domain of a function f is the set of arguments for which f is defined.

The range of a function f is the set of values f can take when applied to an argument.

Domain and range II

If f is a function, then

- the domain of $f = \{x : \text{there is a } y \text{ such that } \langle x, y \rangle \in f\}$,
- the range of $f = \{x : \text{there is a } y \text{ such that } \langle y, x \rangle \in f\}$.

If A is the domain and B the range of f, then f is from A onto B.

If C a superset of the range of f, then f is from A (in)to C.

$$f : A \to B$$

Argument and value

Because of uniqueness, it holds true that for every f applied to x (or f of x), $f(x) :=$ the unique y such that $\langle x, y \rangle \in f$

$f(x) =$ 'the value of f for the argument x'

We say f maps x to y

$$f(x) = y \equiv \langle x, y \rangle \in f$$

Defining functions I

We can list the elements

$$F := \{\langle a, b\rangle, \langle b, c\rangle, \langle c, a\rangle\}$$

$$=$$

$$F := \begin{bmatrix} a \to b \\ b \to c \\ c \to a \end{bmatrix}$$

$$=$$

Let F be that function f with domain $\{a, b, c\}$ such that $f(a) = b, f(b) = c, f(c) = a$.

Defining functions II

Functions with large or infinite domain are defined with conditions.

$F := f : A \to B$
For every $x \in A, f(x) = \ldots$

- A is domain
- B is value
- F is a function from A to B
- F is the function f from A to B such that every argument x in A is mapped by f to the value ...

Defining functions III

Let F_{+1} be that function f such that
$f : \mathbb{N} \to \mathbb{N}$, and for every $x \in \mathbb{N}, f(x) = x + 1$

$=$

$F_{+1} := f : \mathbb{N} \to \mathbb{N}$
For every $x \in \mathbb{N}, f(x) = x + 1$

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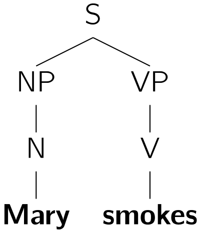
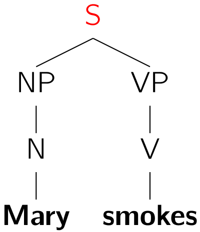
Assumptions about the syntax

Sentences are represented as phrase-structure trees

(4) Mary smokes.



Denotations of sentences and proper names

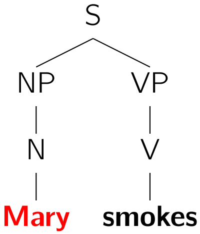


Denotation of S: truth-value – 1 (true) / 0 (false)

Denotation of proper name: individual – Mary

What are the denotations of the remaining nodes?

Denotations of sentences and proper names



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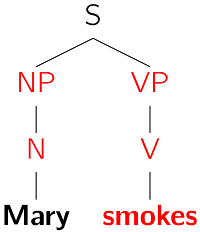

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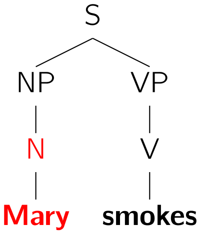
What are the denotations of the remaining nodes?

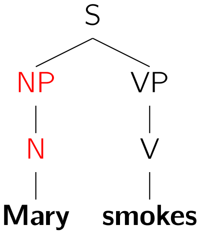
Non-branching nodes in NP



N inherits denotation from **Mary**: Mary

NP inherits denotation from N: Mary

Non-branching nodes in NP



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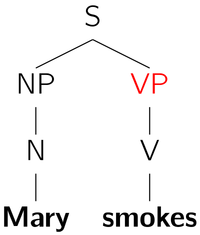
Generalizing to all non-branching nodes

A non-branching node inherits the denotation of its immediate daughter.

Branching nodes



S denotes a truth-value

NP denotes an individual

What does VP denote?

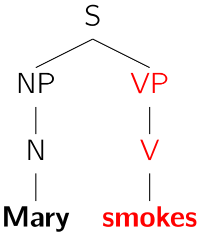
Semantic composition: Frege's conjecture

Semantic composition is function(al) application.

VP must denote a function.

Because of the non-branching node generalization, all of VP's daughters denote that same function.

Nodes in VP



S denotes a truth-value, NP an individual.

VP denotes a function from individuals to truth-values.

V and **smokes** do so, too.

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Component I: inventory of denotations

- Elements of D: set of actual individuals
- Elements of $\{0, 1\}$: set of truth-values
- Functions from D to $\{0, 1\}$
- ...

Component II: lexical entries

- Proper names

  $[\![\textbf{Mary}]\!]$ = Mary
  $[\![\textbf{Bill}]\!]$ = Bill
  ...

- Intransitive verbs

  $[\![\textbf{dances}]\!] = f : D \rightarrow \{0, 1\}$
  For all $x \in D$, $f(x) = 1$ iff x dances
  $[\![\textbf{smokes}]\!] = f : D \rightarrow \{0, 1\}$
  For all $x \in D$, $f(x) = 1$ iff x smokes

$[\![\ ]\!]$ is the interpretation function

$[\![\ ]\!]$ assigns denotation/extension to nodes

$[\![\alpha]\!]$ is the denotation/extension of α in situation s

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Component 3: semantic rules for non-terminal nodes

S1 If α has the form $[_S\ \beta\ \gamma]$, then $[\![\alpha]\!] = [\![\gamma]\!]([\![\beta]\!])$.

S2 If α has the form $[_{NP}\ \beta]$, then $[\![\alpha]\!] = [\![\beta]\!]$.

S3 If α has the form $[_{VP}\ \beta]$, then $[\![\alpha]\!] = [\![\beta]\!]$.

S4 If α has the form $[_N\ \beta]$, then $[\![\alpha]\!] = [\![\beta]\!]$.

S5 If α has the form $[_V\ \beta]$, then $[\![\alpha]\!] = [\![\beta]\!]$.

Component 3: semantic rules for non-terminal nodes

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S
/ \
β γ

S2 If α has the form NP, then $[\![\alpha]\!] = [\![\beta]\!]$.

NP
|
β

S3 If α has the form VP, then $[\![\alpha]\!] = [\![\beta]\!]$.

VP
|
β

S4 If α has the form N, then $[\![\alpha]\!] = [\![\beta]\!]$.

N
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β

S5 If α has the form V, then $[\![\alpha]\!] = [\![\beta]\!]$.

V
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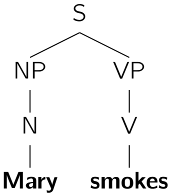
Proof of truth-conditions I



Assume $[\![[_S \ldots]]\!] = 1$ iff Mary smokes

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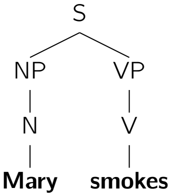

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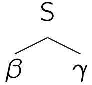

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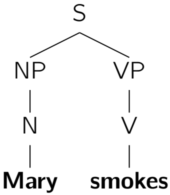

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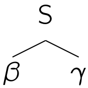

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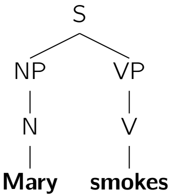

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$\qquad\qquad\qquad\qquad\qquad\;|$
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S4 If α has the form N, then $[\![\alpha]\!] = [\![\beta]\!]$.
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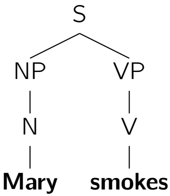

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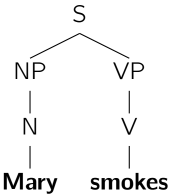

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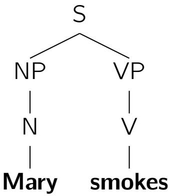

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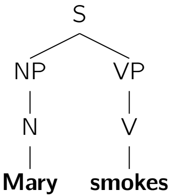
Proof of truth-conditions II



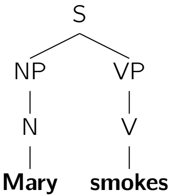
S2 If α has the form $\begin{array}{c}\text{NP}\\ |\\ \beta\end{array}$, then $[\![\alpha]\!] = [\![\beta]\!]$.

$[\![[_S \ldots]]\!] = [\![[_{VP} \ldots]]\!]([\![[_N \ldots]]\!])$

S4 If α has the form $\begin{array}{c}\text{N}\\ |\\ \beta\end{array}$, then $[\![\alpha]\!] = [\![\beta]\!]$.

$[\![[_S \ldots]]\!] = [\![[_{VP} \ldots]]\!]([\![\textbf{Mary}]\!])$

Proof of truth-conditions III



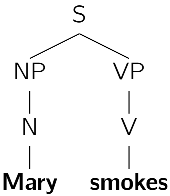
S3 If α has the form $\begin{array}{c}\text{VP}\\ | \\ \beta\end{array}$, then $[\![\alpha]\!] = [\![\beta]\!]$.

$$[\![[_S \ldots]]\!] = [\![[_V \ldots]]\!]([\![\textbf{Mary}]\!])$$

S5 If α has the form $\begin{array}{c}\text{V}\\ | \\ \beta\end{array}$, then $[\![\alpha]\!] = [\![\beta]\!]$.

$$[\![[_S \ldots]]\!] = [\![\textbf{smokes}]\!]([\![\textbf{Mary}]\!])$$

Proof of truth-conditions III



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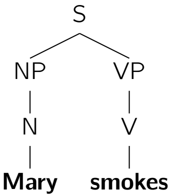

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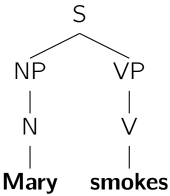

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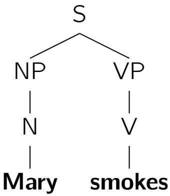

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Proof of truth-conditions IV

$[\![\textbf{Mary}]\!] = \text{Mary}$ lexicon

$[\![\textbf{smokes}]\!] = f : D \to \{0, 1\}$
For all $x \in D$, $f(x) = 1$ iff x smokes lexicon

$[\![[_S \ldots]]\!] = [\![\textbf{smokes}]\!]([\![\textbf{Mary}]\!])$

$$[\![[_S \ldots]]\!] = \left[\begin{array}{l} f : D \to \{0,1\} \\ \text{For all } x \in D, f(x) = 1 \text{ iff } x \text{ smokes} \end{array}\right](\text{Mary})$$

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$[\![[_S \ldots]]\!] = 1$ iff Mary smokes

End of proof

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$[\![[_S \ldots]]\!] = 1$ iff Mary smokes

End of proof

Truth-conditions of a sentence

Lexicon defines extension of an intransitive verb as a condition

$[\![\textbf{smokes}]\!] = f : D \to \{0, 1\}$
For all $x \in D$, $f(x) = 1$ iff x smokes

Together with the rules we get truth-conditions as meaning of S .

This seems adequate regarding the intuitions of speakers with respect to the meaning of $[\![\textbf{smokes}]\!]$.

Normally we do not have full information about who smokes.

Truth-value of a sentence

Assume $D = \{\text{John, Mary, Sue}\}$ in Situation s

Extension of **smokes** in s represented by table

$$[\![\textbf{smokes}]\!] = \begin{bmatrix} \text{John} & \rightarrow & 0 \\ \text{Mary} & \rightarrow & 1 \\ \text{Sue} & \rightarrow & 0 \end{bmatrix}$$

Delivers as denotation of S a truth-value

$$[\![[_S \ldots]]\!] = \begin{bmatrix} \text{John} & \rightarrow & 0 \\ \text{Mary} & \rightarrow & 1 \\ \text{Sue} & \rightarrow & 0 \end{bmatrix}(\text{Mary}) = 1$$

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